APPLICATION OF THE TEMPERATURE WAVE METHOD TO MEASURE THE THERMOPHYSICAL PROPERTIES OF MATERIALS

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A theoretical foundation is given to the methodology for measuring the thermophysical properties of substances and the applicability of the temperature wave method here is analyzed. On the basis of an examination of a two-dimensional heat conduction problem for a disc specimen and a circular modulated thermal flux, the influence of heat transfer on different specimen surfaces is investigated on the temperature wave parameters therein.

As is known, parameters of a temperature weave occurring in a specimen for a certain steady-state quasistationary process are estimated in the periodic heating method [1]. The application of the temperature wave method here discloses the possibility of complex measurement of a number of thermophysical characteristics of the specimen substance.

The theory of the plane temperature wave method [2] is based on a one-dimensional model that, although justifiable in certain cases, does not adequately describe the physical situation as a whole since it is necessary to take account of the inhomogeneity of the thermal flux and the distortion of the isothermal surfaces because of heat transfer during its analysis.

Individual aspects of such an analysis were examined in [3, 4] while it was performed completely by using a two-dimensional model in [5] and with the boundedness of the thermal flux and specimen and heat transfer dimensions on the whole surface of the latter.

The purpose of this paper is to analyze the possibility of applying the temperature wave method for measuring the thermophysical properties of substances.

A radially symmetric problem of heat conduction was considered in [5] for the case of a circular modulated thermal flux and a disc specimen with heat transfer taken into account on all three of its surfaces.

An expression for the complex temperature wave amplitude

$$\Theta(r,\ z) = \frac{2q_0 bL^2 D}{\lambda R^2}, \tag{1}$$

was obtained as a result of solving this problem under the assumption of smallness of the variable temperature wave component in the specimen as compared with the constant component, where r and z are the running coordinates of the problem perpendicular and parallel to the direction of the incident heat flux measured from a central point of the surface upon which it acts

$$D = \sum_{n=1}^{\infty} \frac{\mu_n J_1\left(\mu_n \frac{b}{L}\right) J_0\left(\mu_n \frac{r}{L}\right)\left[\rho_n \operatorname{ch}\left\{\rho_n\left(\frac{z}{L}-1\right)\right\} - \mathrm{Bi}_2 \operatorname{sh}\left\{\rho_n\left(\frac{z}{L}-1\right)\right\}\right]}{[\mathrm{Bi}_3^2+\mu_n^2]\, J_0^2\left(\mu_n \frac{R}{L}\right)[(\mathrm{Bi}_1\mathrm{Bi}_2+\rho_n^2)\operatorname{sh}\rho_n + \rho_n(\mathrm{Bi}_1+\mathrm{Bi}_2)\operatorname{ch}\rho_n]},$$

u_n is a root of the equation

$$\mu_n J_1\left(\mu_n \frac{R}{L}\right) - \mathrm{Bi}_3 J_0\left(\mu_n \frac{R}{L}\right) = 0, \tag{2}$$

J_0, J_1 are Bessel functions, $\rho_n{}^2 = \mu_n{}^2 + i\kappa^2$; $\kappa^2 = \omega/a\ L^2$; and $\mathrm{Bi}_{1,2,3}$ are the Biot numbers, respectively, for the plane surface of heat flux action, the opposite plane surface, and the side surface of the disc specimen.

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We have for the phase shift of the temperature fluctuations at any point of the specimen with respect to thermal flux fluctuations on the surface of its action

$$\varphi = \operatorname{arctg} \frac{\operatorname{Im}\Theta}{\operatorname{Re}\Theta}, \tag{3}$$

while for the amplitude of the temperature fluctuations

$$|\Theta| = \sqrt{(\operatorname{Re}\Theta)^2 + (\operatorname{Im}\Theta)^2}. \tag{4}$$

The expressions (3) and (4) permit determination of a and

$$C_p = \frac{2PLF(\varkappa)}{M|\Theta|\, b\omega}, \tag{5}$$

where $F(\kappa) = \kappa^2\sqrt{(\mathrm{ReD})^2 + (\mathrm{ImD})^2}$, according to the φ and $|\Theta|$ found in experiment. However, the parameters Bi_1, Bi_2, Bi_3 are also unknown as a rule, consequently, in the general case four measurements of φ and $|\Theta|$ must be performed at four different frequencies ω and a system of four equations of the form (3) and one equation of the form (2) must be compiled to determine a, say, and which is solved by numerical methods. The data obtained can then be utilized to determine C_p.

In order to diminish the necessary quantity of measurements performed and to simplify the system of equations compiled later, the influence of Bi_1, Bi_2, Bi_3 on the temperature wave parameters was analyzed. On the basis of the dependences φ, $|\Theta|$ (κ) obtained on an electronic computer for different values of $Bi_{1,2,3}$ and the relationship b/R domains of variation of the problem parameters in which the influence of the heat transfer at one or several specimen surfaces could be neglected and thereby equations to determine the appropriate Biot numbers could be rejected.

We present these results just for the central point (r = 0) of the surface opposite to the surface of heat flux action (z = L). Their dependence on the relations r/R and b/R is detected for deviations from this point along r.

Figure 1 illustrates the case $Bi_1 = Bi_2 = Bi_3 = Bi$. Domains of variation of κ and Bi in which Bi = 0 can be considered with 1% error for the determination of the desired thermophysical characteristics according to φ and $|\Theta|$ found in experiment are located to the right of the displayed boundary lines.

Figure 2 illustrates the case $Bi_1 = Bi_2 \neq Bi_3$. Domains of variation of κ and Bi_3 in which $Bi_3 = 0$ can be considered with 1% error for the determination of the desired thermophysical characteristics according to φ and $|\Theta|$ found in experiment, are located to the right of the displayed boundary lines.

The domain of variation of Bi_1 and Bi_2 in which $Bi_1 = Bi_2$ can be considered with 1% error for the determination of the desired thermophysical characteristics according to the φ and $|\Theta|$ found, is located in Fig. 3 to the left of the displayed boundary line.

The results presented permit making the following deduction. The temperature wave method is applicable for the determination of the thermal diffusivity and specific heat coefficients according to the temperature wave phase and amplitude in the specimen found in experiment, where a sufficiently broad range of variation of the parameters exists in

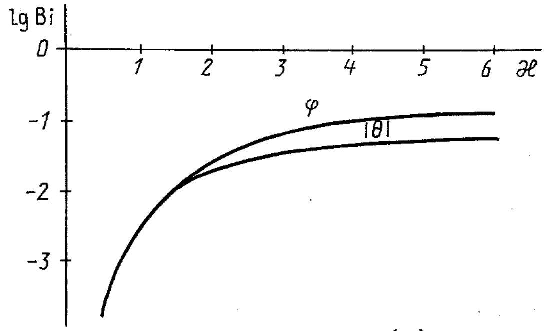

Fig. 1. Boundaries of the domains lg Bi (κ), in which $|[\varphi, |\Theta|(Bi) - \varphi, |\Theta|(0)]|/\varphi, |\Theta|(0) < 0.01$ are satisfied for R/L = 5, L = 0.001, r = 0, b/R = 0.2-1.0

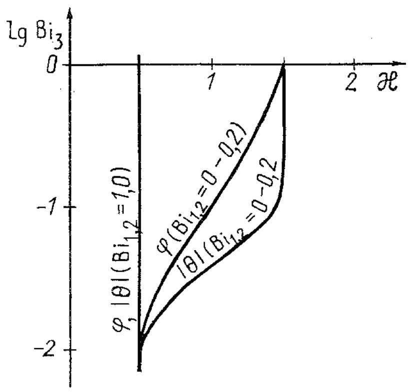

Fig. 2

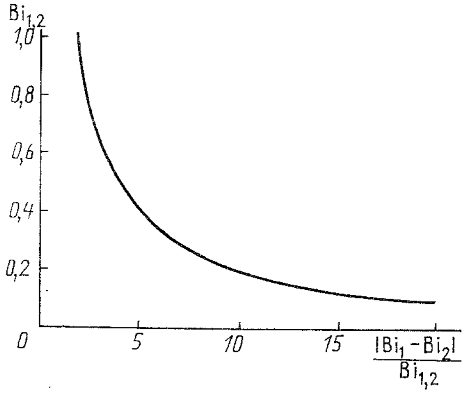

Fig. 3

Fig. 2. Boundaries of the domains of $\lg Bi_3$ (κ) for different values of $Bi_{1,2}$ in which $|[\varphi, |\Theta|(Bi_3) - \varphi, |\Theta|(0)]|/\varphi, |\Theta|(0) < 0.01$ are satisfied for R/L = 5, L = 0.001, r = 0, b/R = 0.2-1.0.

Fig. 3. Boundary of the domain of $Bi_{1,2}$ ($|Bi_1 - Bi_2|/Bi_{1,2}$) in which $|[\varphi, |\Theta|(Bi_1, Bi_2) - \varphi, |\Theta|(Bi_2, Bi_2)]|/\varphi, |\Theta|(Bi_2, Bi_2) < 0{,}01$ and $|[\varphi, |\Theta|(Bi_1, Bi_2) - \varphi, |\Theta|(Bi_1, Bi_1)]|/\varphi, |\Theta|(Bi_1, Bi_1) < 0.01$ are satisfied for R/L = 5, L = 0.001, r = 0, b/R = 0.2-1.0, $Bi_3 = 0 - 2.0$, $\kappa \geq 1.5$.

which the influence of heat transfer on its surface can be neglected and the problem of determining the desired thermophysical characteristics is thereby simplified significantly.

Notation. λ is the coefficient of specimen heat conductivity, W/(m·K); a is the specimen thermal diffusivity coefficient, m^2/sec; C_p is the specimen specific heat, J(kg·K); L is the specimen thickness, m; R is the specimen radius, m; b is the heat flux radius on the action surface, m; q_0 is the heat flux density amplitude, W/m^2; ω is the frequency of heat flux modulation, rad/sec; Bi is the Biot number; M is the specimen mass, kg; P is the power absorbed by the specimen, W; Θ is the complex temperature wave amplitude, K.

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